AP Review Worksheet #3 Name: Key
1997 AB Multiple Choice (No Calculator)

2. If $f(x) = x\sqrt{2x-3}$, then $f'(x) =$ $x \cdot \frac{1}{2}(2x-3)^{-1/2}(2) + (1)\sqrt{2x-3} \cdot \frac{\sqrt{2x-3}}{\sqrt{2x-3}}$

$\frac{x}{\sqrt{2x-3}} + \frac{2x-3}{\sqrt{2x-3}}$

$\frac{3x-3}{\sqrt{2x-3}}$

(A) $\frac{3x-3}{\sqrt{2x-3}}$

(B) $\frac{x}{\sqrt{2x-3}}$

(C) $\frac{1}{\sqrt{2x-3}}$

(D) $\frac{-x+3}{\sqrt{2x-3}}$

(E) $\frac{5x-6}{2\sqrt{2x-3}}$

4) $f'(x) = -3x^2 + 1 - \frac{1}{x^2}$
$f'(-1) = -3 + 1 - 1$
$= -3$

4. If $f(x) = -x^3 + x + \frac{1}{x}$, then $f'(-1) =$

(A) 3 (B) 1 (C) −1 (D) −3 (E) −5

5. The graph of $y = 3x^4 - 16x^3 + 24x^2 + 48$ is concave down for

$y' = 12x^3 - 48x^2 + 48x$
$y'' = 36x^2 - 96x + 48 \rightarrow 0 = 12(3x^2 - 8x + 4)$
$(3x-2)(x-2)$

f'' : 0, 1, 3 ; $\frac{2}{3}$, 2 ; (+) (−) (+) ; conc. dwn.

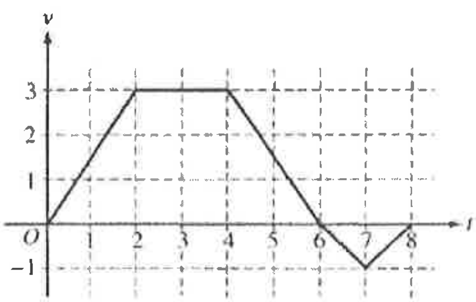
(A) $x < 0$

(B) $x > 0$

(C) $x < -2$ or $x > -\frac{2}{3}$

(D) $x < \frac{2}{3}$ or $x > 2$

(E) $\frac{2}{3} < x < 2$

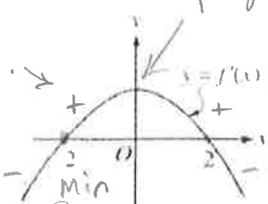


A bug begins to crawl up a vertical wire at time $t = 0$. The velocity v of the bug at time t, $0 \le t \le 8$, is given by the function whose graph is shown above.

8. At what value of t does the bug change direction? when velocity goes from + to − @ t=6

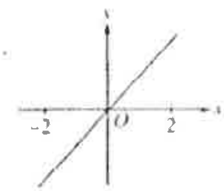
(A) 2 (B) 4 (C) 6 (D) 7 (E) 8

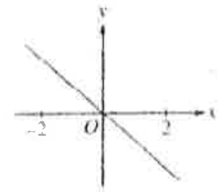
f' goes from incr to decr.) f'' + to −
PI

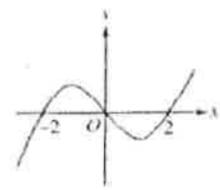
f incr. f incr


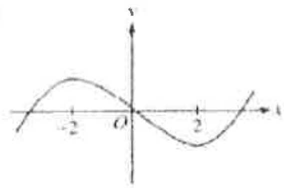
Max @ $x = 2$
f decr → f decr.
Min @ $x = -2$

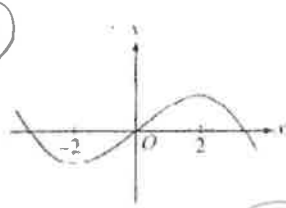
11. The graph of the derivative of f is shown in the figure above. Which of the following could be the graph of f?

(A)


(B)


(C)


(D)


(E)


slope $y' = 2\left(\frac{1}{2}x\right)$ → where is it $= \frac{1}{2}$?
$y' = x$
$\frac{1}{2} = x$ → on parabola $y = \frac{1}{2}\left(\frac{1}{2}\right)^2 = \frac{1}{8}$

12. At what point on the graph of $y = \frac{1}{2}x^2$ is the tangent line parallel to the line $2x - 4y = 3$? → $y = \frac{-2x}{-4} + \frac{3}{-4}$

$m = \frac{1}{2}$

(A) $\left(\frac{1}{2}, -\frac{1}{2}\right)$ (B) $\left(\frac{1}{2}, \frac{1}{8}\right)$ (C) $\left(1, -\frac{1}{4}\right)$ (D) $\left(1, \frac{1}{2}\right)$ (E) $(2, 2)$

13. Let f be a function defined for all real numbers x. If $f'(x) = \frac{|4 - x^2|}{x - 2}$, then f is decreasing on the interval

where f' is negative; $\frac{+}{-}$
so $x - 2 < 0$
$x < 2$

(A) $(-\infty, 2)$ (B) $(-\infty, \infty)$ (C) $(-2, 4)$ (D) $(-2, \infty)$ (E) $(2, \infty)$

14. Let f be a differentiable function such that $f(3) = 2$ and $f'(3) = 5$. If the tangent line to the graph of f at $x = 3$ is used to find an approximation to a zero of f, that approximation is

→ $y - 2 = 5(x - 3)$
x-int:
$0 - 2 = 5x - 15$
$13 = 5x$
$x = \frac{13}{5} \approx 2.6$

slope = 5 at $x = 3$
$(3, 2)$
zero btwn 2 & 3
$(2, -3)$

(A) 0.4 (B) 0.5 (C) 2.6 (D) 3.4 (E) 5.5

17. If $x^2 + y^2 = 25$, what is the value of $\frac{d^2y}{dx^2}$ at the point $(4, 3)$? $2x + 2y\frac{dy}{dx} = 0$

$\frac{dy}{dx} = -\frac{x}{y}$

$\frac{d^2y}{dx^2} = \frac{y(-1) - (-x)\frac{dy}{dx}}{y^2}$

$= \left[-y + x\left(-\frac{x}{y}\right)\right]\frac{1}{y^2}$

$= -\frac{y^2}{y^3} - \frac{x^2}{y^3}$

$= -\frac{(x^2 + y^2)}{y^3}$

$= -\frac{25}{y^3}$ ← 3^3

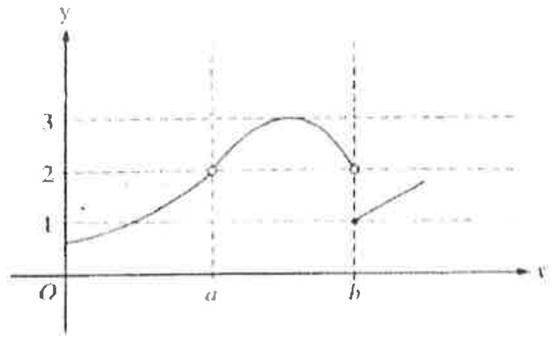
(A) $-\frac{25}{27}$ (B) $-\frac{7}{27}$ (C) $\frac{7}{27}$ (D) $\frac{3}{4}$ (E) $\frac{25}{27}$

21. $\lim_{x \to 1} \frac{x}{\ln x}$ is

← $\ln 1 = 0$ ‖

so $\lim_{x \to 1} \frac{1}{0} = \infty$ DNE

(A) 0 (B) $\frac{1}{e}$ (C) 1 (D) e (E) nonexistent



15. The graph of the function f is shown in the figure above. Which of the following statements about f is true?

~~(A)~~ $\lim_{x\to a} f(x) = \lim_{x\to b} f(x)$ ← outcomes approach 2 different values

(B) $\lim_{x\to a} f(x) = 2$

~~(C)~~ $\lim_{x\to b} f(x) = 2$ ← only from the left.

(D) $\lim_{x\to b} f(x) = 1$ ← from the right

~~(E)~~ $\lim_{x\to a} f(x)$ does not exist.

limit def. of the derivative. pg 98

Calculator ok

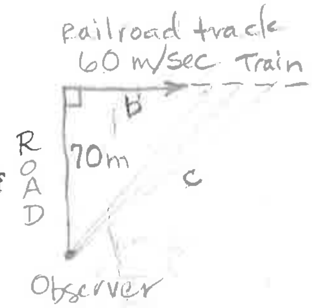
79. Let f be a function such that $\lim_{h\to 0} \frac{f(2+h)-f(2)}{h} = 5$. Which of the following must be true?

Since the limit exists, f must be differentiable and continuous @ $x=2$

I. f is continuous at $x = 2$.
II. f is differentiable at $x = 2$. } Must be true
III. The derivative of f is continuous at $x = 2$. ← ?

(A) I only (B) II only (C) I and II only (D) I and III only (E) II and III only

81. A railroad track and a road cross at right angles. An observer stands on the road 70 meters south of the crossing and watches an eastbound train traveling at 60 meters per second. At how many meters per second is the train moving away from the observer 4 seconds after it passes through the intersection?



(A) 57.60 (B) 57.88 (C) 59.20 (D) 60.00 (E) 67.40

Find $\frac{dc}{dt}$, when $t=4$

$b = \frac{60m}{sec} \cdot 4\,sec = 240\,m$

$c = \sqrt{240^2 + 70^2}$

$c = 250$

Related Rates Equation

$a^2 + b^2 = c^2$

$\cancel{2}a\frac{da}{dt} + \cancel{2}b\frac{db}{dt} = \cancel{2}c\frac{dc}{dt}$

$70(0) + (240)(60) = 250\frac{dc}{dt}$

No change in position of observer

$\frac{dc}{dt} = \frac{240(60)}{250}$

$= 57.6\ m/sec$

82. If $y = 2x - 8$, what is the minimum value of the product xy? See below,

(A) −16 (B) −8 (C) −4 (D) 0 (E) 2

86. Let $f(x) = \sqrt{x}$. If the rate of change of f at $x = c$ is twice its rate of change at $x = 1$, then $c =$

(A) $\frac{1}{4}$ (B) 1 (C) 4 (D) $\frac{1}{\sqrt{2}}$ (E) $\frac{1}{2\sqrt{2}}$

rate of change f

$f'(x) = \frac{1}{2}x^{-1/2}$

$f'(x) = \frac{1}{2\sqrt{x}}$

$f'(c) = 2f'(1)$

$\frac{1}{2\sqrt{c}} = 2\left(\frac{1}{2\sqrt{1}}\right)$

$\sqrt{c}\,\frac{1}{2\sqrt{c}} = 1\,\sqrt{c}$

$\left(\frac{1}{2}\right)^2 = (\sqrt{c})^2$

$c = \frac{1}{4}$

82) $f(x) = xy$
$f(x) = x(2x-8)$
$f(x) = 2x^2 - 8x$
Min where $f' = 0$ & $f'' = +$
$f'(x) = 4x - 8$
$0 = 4x - 8$
$x = 2$ $f''(x) = 4$ (+)
Min value is the y-value @ $x=2$ Confirms Min
$f(2) = 2(4-8) = -8$